DESARROLLO Y SOCIEDAD 58
SEGUNDO SEMESTRE DE 2006, PP. 1-35.

Addiction and Self-Control: An Intrapersonal Game

Adicción y autocontrol: un juego intrapersonal

Rafael López*

Abstract

In their model of addiction, O'Donoghue and Rabin obtain a counter-intuitive result: a person that is fully aware of his self-control problems (sophisticate) is more prone to become addicted than one who is fully unaware (naïf). In this paper we show that this result arises from their particular equilibrium selection for the induced intra-personal game. We provide dominating Markov Perfect equilibria where the paradox vanishes and that seem more "natural" since they capture behaviors often observed in the realm of addiction. We also address the issue of why an unaddicted person could decide to start consuming and possibly develop an addiction. In particular, we show that their equilibrium implies that both naifs and sophisticates will slip into addiction. In contrast, by considering our results, only naifs will become addicted which is in accordance to the common intuition. Finally, we suggest a clear-cut way

*I am grateful to Fernando Vega-Redondo, Antonio Cabrales and Fabrizio Germano for helpful comments; and to workshop participants at Universitat Pompeu Fabra, Universitat d'Alacant and the Central European University.

Visiting Professor, Department of Economics, Complutense University of Madrid, Campus de Somosaguas 28223 Pozuelo de Alarcón, Madrid, Spain. Email: *ralopez@ccee.ucm.es*.

This paper was received April 10, 2006, modified October 30, 2006 and accepted December 1, 2006.

of modeling partial awareness of self-control problems; a topic that has received little attention in the literature.

Key words: addiction, self-control, negative internalities, habit formation, hyperbolic discounting, naïveté, sophistication, time inconsistency.

JEL Classification: A12, C79, D11, D60, D91.

Resumen

En su modelo de adicción, O'Donoghue y Rabin obtienen un resultado contra-intuitivo: un individuo plenamente conciente de sus problemas de autocontrol (i.e. un sofisticado) es más propenso a caer en la adicción que un individuo que los ignora por completo (i.e. un naïf). En este artículo se muestra que dicho resultado es una consecuencia directa de la particular selección del equilibrio que realizan para el juego intrapersonal inducido. Nosotros proporcionamos equilibrios Perfectos de Markov donde la paradoja desaparece y que resultan más "naturales" pues reflejan comportamientos comúnmente observados en los contextos de adicción. También se aborda la razón por la cual una persona que se inicia en el consumo de una sustancia adictiva posiblemente cae en la adicción. En particular, se muestra que bajo el equilibrio seleccionado por O&R tanto los naïfs como los sofisticados caerían en la adicción mientras que considerando nuestros equilibrios, sólo los naïfs sucumbirían. El resultado está claramente más de acuerdo con el sentido común. Finalmente se sugiere una forma simple de modelar la conciencia parcial de los problemas de autocontrol, contribuyendo así a un tema que hasta el momento ha recibido muy poca atención en la literatura.

Palabras clave: adicción, autocontrol, internalidades negativas, formación de hábito, descuento hiperbólico, naïveté, sofisticación, inconsistencia temporal.

Clasificación JEL: A12, C79, D11, D60, D91.

Introduction

Mr. X goes to a party where he is offered a pill - call it Panacea -. He knows for sure that if he takes it he will experience immediate pleasure but he is also aware that some of his neuronal cells will pay for his decision. Since most humans, and particularly X, use but a small fraction of their brain capacity, he might as well give up those neuronal cells without experiencing a significant loss. However, he is also aware that pill would lead to some more which altogether will produce a certain brain clash. Should I stay or should I go? X asked to himself. As the indecision bothered him, he evaluated whether the immediate pleasure offset the future brain damage and proceeded accordingly.

We may not know whether X took Panacea or not, but we certainly know that he is a rational forward-looking person: when adopting his decision he knew the future consequences of his choice. Mr. X was perfectly aware of the two characteristics constituting the crux of an addictive substance, namely, the *habit-forming* property (present pill raising future consumption); and the *negative internalities* induced by consumption (present pill reducing future well-being, via the brain clash).

In their famous work, Becker and Murphy (1988) modeled consumption of a good presenting these two features as a rational process where addiction is understood as the outcome of intended behavior (i.e., intertemporal utility maximization) under perfect foresight. In particular, their Rational Addiction model implies that an addict does not regret his previous decisions and perfectly forecasts his future consumption; two elements that have largely been criticized (surveys of these critics are found in Chaloupka and Warner 1998 and Messinis 1999). On psychological grounds, addiction certainly entails planned behavior but it also involves self-control problems that give rise to regret and misprediction of future conduct. This is clearly illustrated by Heyman (1996):

> Drug consumption is a goal oriented act. The behaviors are learned, not reflexive or innate. It requires planning, effort, and in some cases artfulness to secure drugs in the amount necessary for maintaining an addiction. Yet, according to the diagnostic

> manuals (e.g., DSM-III-R and ICD-10), the feature that defines addiction is drug use which is 'out of control' or 'compulsive'. By these phrases, the manuals mean that addicts 'take more drug that they initially intended', that drug use persists despite a wide array of ensuing legal, medical, and social problems, and that after periods of abstinence, however long, addicts relapse.

As Gruber and Koszegi (2001) point out, "The term 'rational addiction' obscures the fact that the Becker and Murphy model imposes two assumptions on consumer behavior. The first is that of forward-looking decision-making, which is hard to impugn (...). The second is the assumption that consumers are time consistent. Psychological evidence documents overwhelmingly that consumers are time inconsistent" (page 16).

Recently and in different contexts, many economists have studied self-control problems modeling them in terms of the time inconsistency derived from non-exponential discounting[1]. Supported by empirical evidence showing that subjects exhibit declining discount rates (e.g. Thaler (1991); Loewenstein and Prelec (1992)), most of these studies use hyperbolic discounting (for an excellent review on hyperbolic discounting and time preference see Frederick, Loewenstein and O'Donoghue (2002)).

O'Donoghue and Rabin (2002) (from now on O&R) combine the Becker and Murphy approach with hyperbolic discounting[2] in their modeling of

[1] This approach to model self-control problems derives from the pioneering work by Strotz (1956) who noted that when using a non-exponentional discount function intertemporal utility gives rise to time inconsistency in the sense that an optimal plan at some particular date may no longer be optimal at further dates. However, self-control problems may also be modeled while maintaining time consistency (i.e. exponential discounting). For instance, Laibson (2001) and Bernheim and Rangel (2003) model self-control problems by introducing cue-conditioned behavior. In their models, environmental cues may trigger a "hot" mode in which the individual consumes the addictive substance disregarding its future consequences (i.e. she "looses control").

[2] The specific discounting functional form they use (which we formally present in Section I) is not really hyperbolic but it captures the essence of hyperbolic discounting, namely, present-biased preferences. It was first introduced by Phelps and Pollack (1968) and because of its simplicity and tractability, it has been widely used to model self-control problems since the work of Laibson (1994).

addiction. In their framework, an infinitely lived individual[3] has to decide at each period whether to consume or not a free addictive product; i.e. a product presenting the habit-forming and negative internalities features. As in the Becker-Murphy model, the individual is perfectly aware of these two features. But due to the time inconsistent preferences embodied in hyperbolic discounting the individual may not be able to follow his optimal consumption path thus giving rise to self-control problems. Concerning the awareness of these problems, they distinguish two extreme types of individuals: naifs, who are totally unaware; and sophisticates, who are perfectly aware. A naif believes that his future selves will follow his optimal consumption path thus choosing his current action accordingly. But because of his time inconsistent preferences, his future selves will often revise the optimal plan hence yielding a different path from the one intended. As a consequence, a naïf usually falls in over-consumption (note that this result captures Heyman's description "addicts take more drug that they initially intended"). A sophisticate knows that the optimal consumption path he is aiming at may be revised by his future selves and thus may not be followed. Therefore he chooses his current action according to the best path that can be pursued by his future selves. In a sense, a sophisticate is playing an intrapersonal game: he plays against his future selves. The solution concept they propose is that of *perception-perfect strategy equilibrium*[4] (from now on we will refer to it as the ORE). However, the ORE has the shortcoming of producing a counterintuitive result: under some circumstances, sophisticates will consume always (i.e. become addicted) while naifs might not. As they point out, this "contradicts the common intuition that harmful addictions are caused by people naively slipping into an unplanned addiction". Following O&R we will refer to those circumstances as the *inevitability condition* (IC from now on).

[3]They also treat the case of an individual with finite horizon but mainly as a means to understand the infinite horizon case. Indeed, in the context of addiction, an infinite horizon seems a much better approximation of real behavior.

[4]In the induced game with a finite horizon T, there is a unique subgame perfect equilibrium; call it T-equilibrium. In the infinite horizon case, a perception-perfect strategy equilibrium is simply the limit of the sequence of T-equilibria as T becomes long.

In the present paper we show first that this counterintuitive result is obtained by their particular equilibrium selection (ORE) and that there are more "natural" dominating equilibria where the paradox vanishes; i.e. where sophisticates are less prone to become addicted than naifs. Since in an intrapersonal game the players are just incarnations of the *same* individual, coordination on a dominated equilibrium cannot be supported and therefore we argue that the ORE is not the appropriate selection. Secondly, we address the issue of *developing an addiction,* that is, we analyze the circumstances under which an unaddicted person could decide to start consuming and whether she could become addicted or not. In particular, we show that the ORE solution is of no use for studying this issue since it implies that both naifs and sophisticates will slip into addiction. In contrast, by considering our results, naifs will become addicted while sophisticates will not which is in accordance to the common intuition cited above. Finally, we suggest a very clear-cut way of modeling partial awareness of self-control problems.

The importance of our findings can be motivated in terms of policy implications. Consider for example a public advertising campaign providing information on self-control problems induced by drug consumption. What such a campaign would normally do is a shift from naiveness to sophistication given that people become aware of their time-inconsistency. Under our results such a campaign would be successful in reducing addiction (since sophisticates are less prone to become addicted than naifs) while under the O&R result it would produce the opposite effect. Wide existence of such campaigns favors our results.

The paper proceeds as follows. In Sections I and II we formally present the O&R model and their results, stating clearly under which circumstances the paradox is obtained. In Section III we study equilibria in cutoff strategies by providing a complete characterization: in particular, we state conditions under which the ORE generates the paradox and yet there is a dominating cutoff equilibrium that solves it. But cutoff equilibria may not exist or may not solve the paradox, therefore, in Section IV, we provide non-cutoff dominating equilibria which solve it whenever generated. In Section V we address the issue of developing an addiction and argue that the ORE solution fails to explain this issue while our results prove to be in accordance to the common intuition. Section VI concludes and suggests a "natural" way of modeling partial awareness of self-control problems, a topic that so far has received very little attention in the literature.

I. The O&R Model

In the O&R model, an individual decides at each period t, whether to consume (hit) or not (refrain) a free addictive product. Let a_t be the binary variable reflecting the individual's choice at time t: $a_t = 1$ meaning he decides to hit whereas $a_t = 0$ means he decides to refrain.

His period-t instantaneous utility is given by

$$\forall t, u\left(k_t, a_t\right) = \begin{cases} x + f\left(k_t\right) & \text{if} \quad a_t = 1 \\ g\left(k_t\right) & \text{if} \quad a_t = 0 \end{cases} \tag{1}$$

where k_t is the individual's level of addiction which captures all the effects of past consumption on current instantaneous utility. The level of addiction is assumed to evolve according to $k_{t+1} = \gamma k_t + a_t$ with $0 < \gamma < 1$. Therefore, there is a maximal addiction level $k^{\max} = \frac{1}{1-\gamma}$. Note that instantaneous utility is stationary in the sense that it depends on the prevailing level of addiction at period t but not on the particular period t. Addiction is modeled by making the following assumptions on f, g and x :

> **Assumption 1:** $f', g' < 0$. This assumption introduces the feature of negative internalities since the more a person has consumed in the past (as captured by his addiction level) the lower his current instantaneous utility. Without loss of generality, it is assumed $f\left(0\right) = g\left(0\right) = 0$.
>
> **Assumption 2:** $f' - g' > 0$. This assumption introduces the habit-forming feature. To see this, let $h(k) = x + f(k) - g(k)$ be the temptation to hit (i.e. the marginal instantaneous utility of hitting). Then $h'\left(k\right) > 0$ implies that hitting is more desirable the higher the level of addiction; i.e. past consumption of the product (as captured by

the addiction level) increases current desire for consumption.

Assumption 3: $f'', g'' \geq 0$. In addition to negative internalities and habit-forming, it is assumed that the more addicted a person becomes the less a given increase in k hurts his instantaneous utility, and therefore less harm hitting induces in future utility.

Assumption 4: $x > 0$. This assumption says that the temptation to hit is positive even for an unaddicted person.

Self-control problems are modeled by assuming present-biased preferences as in the Phelps and Pollak intertemporal utility function given by:

$$U(u_t, u_{t+1}, ..., u_T) = u_t + \beta \sum_{\tau=t+1}^{T} \delta^{\tau-t} u_\tau \text{ with } \beta \in (0,1)$$

where each u_τ is the period-τ instantaneous utility given by (1) and where the parameter β introduces the present bias. O&R consider both general cases: when the individual faces a finite horizon ($T < \infty$) and an infinite horizon ($T = \infty$). We will consider only the infinite horizon version because we believe that it is more realistic: assuming a finite horizon would imply that the individual knows in advance the last period of his life. As it is standard in Game Theory, the use of an infinite horizon in a dynamic or repeated game constitutes a better approximation to a realm where the "last period" is unknown. Moreover, in this setting the parameter δ may be interpreted as the probability of surviving one period. Therefore, the intertemporal utility function we will use takes the form:

$$U(u_t, u_{t+1}, ...) = u_t + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u_\tau \text{ with } \beta \in (0,1) \tag{2}$$

Before stressing out the implications of (2) it is useful to consider the case of a typical intertemporal utility function with exponential discounting, i.e. (2) with $\beta = 1$. Following O&R we will refer to a rational forward-looking person having such preferences as a time consistent individual (TC).

A. Time Consistent Individuals (TC)

A TC's preferences are given by

$$U(u_t, u_{t+1}, ...) = u_t + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u_\tau \tag{3}$$

Definition 1 *A behavior path $A = (a_1, a_2, ...)$ is an infinite sequence of admissible actions; i.e. $\forall i, a_i \in \{0, 1\}$.*

Three particular behavior paths are of special interest and we will label them as follows: the hitting path $H = (1, 1, ...)$; the refraining path $R = (0, 0, ...)$; and the hitting once path $O = (1, 0, 0, ...)$. Let $U_{tc}^A(k_t)$ denote the intertemporal utility given in (3) associated to following the behavior path A from an initial addiction level k_t (the stationary instantaneous utility function implies that the unique payoff relevant variable at any date t is the prevailing addiction level). Being rational forward-looking amounts to saying that at any given period t a TC solves

$$\max_{A \in \{0,1\}^\infty} U_{tc}^A(k_t) \tag{4}$$

and then chooses the first action corresponding to the solution path . We will refer to such a solution as a desired behavior path (DBP). For a TC there is time consistency: for any starting addiction level k_t a DBP at some date t is still optimal at any further date. Therefore a TC has no self-control problems since future selves have no incentives to deviate from a DBP chosen by a previous self.

O&R show that under stationary instantaneous utility there exists a critical addiction level k^{tc} such that each self solves (4) by choosing to hit if and only if $k_t \geq k^{tc}$. As a consequence, a TC's DBP is either hitting always or refraining always. We state this result as a proposition:

Proposition 2 $\exists k^{tc} \in [0, k^{\max}]$ *such that the DBP for a TC with starting addiction level k is H if $k \geq k^{tc}$and R otherwise.*

We turn now to study the consequences of the preferences given by (2).

B. Individuals with Time Inconsistent Preferences (TI): Naifs and Sophisticates

Let $U^A(k_t)$ be the intertemporal utility given in (2) associated to following the behavior path A from an initial addiction level k_t . A rational forward-looking individual with time inconsistent preferences aims at solving

$$\max_{A\in\{0,1\}^\infty} U^A(k_t) \tag{5}$$

However, in this case there is time-inconsistency: a DBP (a behavior path solving (5)) at date t may no longer be optimal at a further date, in the sense that future selves may have incentives to deviate from it thus giving rise to self-control problems. O&R distinguish two types of individuals with preferences induced by (2): Naifs, who are totally unaware of their time-inconsistency; and Sophisticates who are fully aware of their time-inconsistency. A Naif believes that he has no self control problems, that is, he believes that any optimal plan he chooses will be followed by his future selves. Thus, at any given period, a naif simply chooses his current action according to the path solving (5), but the chosen path may be systematically revised at further periods. A Sophisticate is perfectly aware of his self-control problems, he knows that the path he is aiming at may be revised by his future selves and thus may not be followed. Therefore the best he can do is to maximize (5) subject to the condition that the chosen path will be followed by his future selves. A sophisticate is thus playing an intrapersonal game where his opponents are his future selves.

We turn now to study the DBP for a TI. First notice that a TI would like to behave like a TC from next period on . Therefore, given Proposition 2, a TI's DBP (i.e. a path A solving (5)) must involve either hitting always or refraining always from next period on.

This leaves us with only four possibilities for the DBP, namely H, R, O or $(0, 1, 1, ...)$. However we can discard the last one. The intuition is simple: if the individual knows that he will become addicted from tomorrow on, there is no sense in refraining today. The formal proof is given in the following Lemma.

Lemma 3 $A = (0, 1, 1, ...)$ *cannot be the DBP for a TI.*
Proof. *Suppose* $\exists k$ *such that* A *is the respective DBP. Then it must be true that* $k > \gamma k \geq k^{tc}$ *and therefore* $U_{tc}^{H}(k) > U_{tc}^{A}(k)$. *But then*

$$U_{tc}^{H}(k) = u(1,k) + \delta U_{tc}^{H}(\gamma k + 1) > u(0,k) + \delta U_{tc}^{H}(\gamma k) = U_{tc}^{A}(k)$$

which implies

$$u(0,k) - u(1,k) < \delta\left(U_{tc}^{H}(\gamma k + 1) - U_{tc}^{H}(\gamma k)\right) \leq \beta\delta\left(U_{tc}^{H}(\gamma k + 1) - U_{tc}^{H}(\gamma k)\right)$$

where the last inequality follows from $U_{tc}^{H}(\gamma k + 1) - U_{tc}^{H}(\gamma k) \leq 0$. *Therefore*

$$U^{H}(k) = u(1,k) + \beta\delta U_{tc}^{H}(\gamma k + 1) > u(0,k) + \beta\delta U_{tc}^{H}(\gamma k) = U^{A}(k)$$

A contradiction.

Lemma 3 implies the following proposition.

Proposition 4 *For any given starting addiction level* k, *the DBP of a TI admits only one of the following possibilities (we assume he hits when indifferent):* H, R *or* O.

Proposition 5 *Let* A *be any behavior path. Then*

1. $U^{A}(k)$ is decreasing.
2. $\forall k$, $\frac{\partial U^{H}(k)}{\partial k} \geq \frac{\partial U^{A}(k)}{\partial k} \geq \frac{\partial U^{R}(k)}{\partial k}$

Part 1 follows directly from negative internalities while part 2 obtains mainly from the habit-forming assumption (it also requires convexity of f and g or at least them being not too much concave). Note that, in particular, part 2 implies $\frac{\partial U^{H}(k)}{\partial k} \geq \frac{\partial U^{O}(k)}{\partial k} \geq \frac{\partial U^{R}(k)}{\partial k}$. The formal proofs are given in the appendix.

Following O&R, we will define now three important levels of addiction:

- k^{HR}: addiction level such that always hitting is preferred to always refrain if and only if $k \geq k^{HR}$. Formally, let $\widetilde{k}$ be the solution to $U^{H}(k) = U^{R}(k)$, then $k^{HR} = \max\left\{0, \widetilde{k}\right\}$ follows from Proposition 5, part 2.

- k^{OR}: addiction level such that hitting once is preferred to always refrain if and only if $k \geq k^{OR}$. Formally, let $\widetilde{k}$ be the solution to $U^O(k) = U^R(k)$, then $k^{OR} = \max\left\{0, \widetilde{k}\right\}$ follows from Proposition 5, part 2.

- k^{HO}: addiction level such that hitting always is preferred to hitting once if and only if $k \geq k^{HO}$. Formally, let $\widetilde{k}$ be the solution to $U^H(k) = U^O(k)$, then $k^{HO} = \max\left\{0, \widetilde{k}\right\}$ follows from Proposition 5, part 2

Remember that the law of motion of k implies a maximum addiction level $k^{\max} = \frac{1}{1-\gamma}$. According to the formal definitions of k^{HR}, k^{OR} and k^{HO} it could be the case that some of them are above $k^{\max}$. We will say that k^{HR}, k^{OR} and k^{HO} exist if all of them are below $k^{\max}$. Because of Proposition 5, part 2, existence of k^{HR}, k^{OR} and k^{HO} is equivalent to requiring $U^H(k^{\max}) \geq U^O(k^{\max}) \geq U^R(k^{\max})$. We will assume throughout that this condition holds.

As O&R point out, in general, k^{HR} and k^{OR} are not rankable so we will usually distinguish two cases as shown in Figure 1.

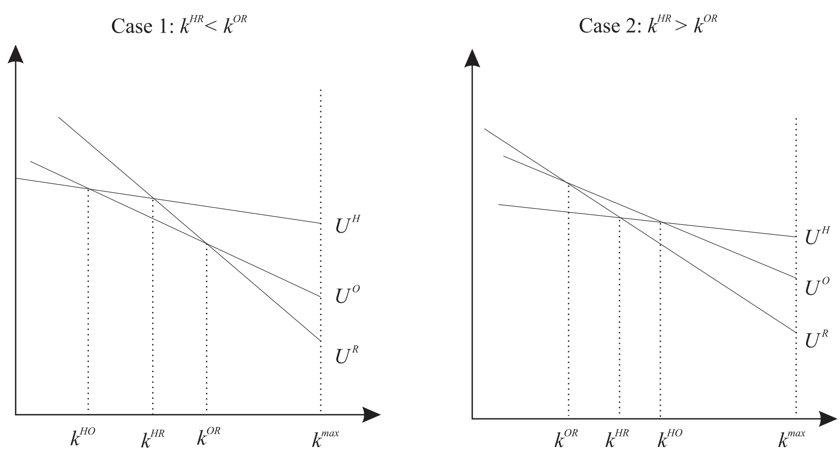


Figure 1: k^{HR} and k^{OR} are not rankable

From Propositions 4 and 5 we directly obtain the characterization for a TI's DBP:

Proposition 6 *a TI's DBP is*

1. R; for any $k < \min\left\{k^{HR}, k^{OR}\right\}$
2. H; for any $k \geq \max\left\{k^{HR}, k^{HO}\right\}$
3. O; for any $k \in \left[k^{OR}, k^{HO}\right)$

We close this section by distinguishing the role played by addiction and present-biased preferences in Figure 1 and Proposition 6: while addiction determines the decreasing intertemporal utility functions and their relative slopes (as depicted in both cases of Figure 1), present-biased preferences induce the possibility of path O being the DBP (Case 2 in Figure 1, Point 3 in Proposition 6).

II. The O&R Results

In the previous section we established the DBP for both TC and TI individuals. We are interested now in determining the realized behavior path (RBP), that is, the path actually followed for each type of individuals. This amounts to specifying the actions to be undertaken by an individual in any particular situation. Because of the stationarity of utility functions and the infinite horizon it seems natural to make those actions time-independent: at a particular date, the action of an individual should depend only on the prevailing addiction level since this is the only payoff relevant variable; the calendar time is irrelevant. Since both a TC (correctly) and a Naif (wrongly) believe that they are able to follow their respective DBP, O&R show that they implement cutoff actions. We state their results in the following propositions.

Proposition 7 *Let $\alpha^{tc}(k)$ be the action taken by a TC when his addiction level is k. Then, $\exists k^{tc} \in [0, k^{\max}]$ such that $\alpha^{tc}(k) = 1 \iff k \geq k^{tc}$. Therefore, the RBP of a TC is either H or R.*

Proposition 8 *Let $\alpha^n(k)$ be the action taken by a Naif when his addiction level is k. Then, $\alpha^n(k) = 1 \iff k \geq \min\left\{k^{OR}, k^{HR}\right\} = k^n$. Therefore, the RBP of a Naif is either H or R.*

Proposition 7 comes directly from Proposition 2. Proposition 8 comes from the fact that a naif, believing that he is going to be able to follow his DBP, will decide to hit if and only if his DBP is either H or O. But this happens if and only if $k \geq \min\left\{k^{HR}, k^{OR}\right\} = k^n$. O&R also show that $k^n \leq k^{tc}$, an intuitive result since a naif discounts the future at a higher rate than a TC and therefore the future harm of hitting is lower for a naif than for a TC.

Let's turn now to the sophisticate case. Because of his awareness of self-control problems, a sophisticate is involved in strategic considerations. The natural solution concept to be called upon for the sophisticate's intrapersonal game is that of Markov Perfect Equilibrium (MPE). Among the multiple MPE for the infinite horizon case, O&R only consider the one corresponding to the limit of the unique finite-horizon MPE as the horizon becomes long. From now on we will refer to this equilibrium as the ORE. The RBP generated by this particular equilibrium selection depends heavily on whether the following condition is satisfied or not.

ORE and the Inevitability Condition (IC)

We say that IC holds if and only if $U^H(0) \geq U^{(0,1,1,\ldots)}(0)$, i.e. whenever an unaddicted individual prefers hitting always to refraining today and hitting thereafter. As we will see later on, this condition implies that in the ORE the individual will decide to hit always, hence the idea of addiction being inevitable. Let $\alpha^s(t, k)$ denote the strategy played by a sophisticated self-t in the ORE. Notice that we are allowing for the strategy to depend on the particular period t. This is so because with a finite horizon the strategy usually depends on the prevailing period and nothing ensures us that when taking the limit as the horizon becomes long we obtain a time-independent strategy. O&R completely characterize the ORE when IC holds and partially when it is not satisfied. We state their results in the following proposition.

Proposition 9 *Partial Characterization of the ORE.*

1. If IC holds then $\forall t, k, \alpha^s(t,k) = 1$; i.e. the sophisticate's RBP in the ORE is H.

2. If IC is not satisfied then

   (a) If $\gamma k^{OR}+1 \geq k^{HR}$ then $\alpha^s(t,k) = 1$ if and only if $k \geq k^{HR}$

   (b) If $\gamma k^{OR}+1 < k^{HR}$ then $\alpha^s(t,k) = \begin{cases} 0 & \text{if} \quad k < k^{OR} \\ ? & \text{if} \quad k^{OR} \leq k < k^{HR} \\ 1 & \text{if} \quad k \geq k^{HR} \end{cases}$

Two striking features of the ORE are to be mentioned. Concerning part 1 notice that when IC holds a sophisticate is more prone to become addicted than a naif since a sophisticate will always hit while a naif might not (given Proposition 8, a naif will always hit if and only if $k \geq \min\{k^{HR}, k^{OR}\}$). As O&R point out, this "contradicts the common intuition that harmful addictions are caused by people naively slipping into an unplanned addiction". However, we claim that this counterintuitive result is obtained by the particular equilibrium selection proposed by O&R (i.e. the ORE) and that it vanishes when considering other type of MPE. Moreover, we claim that there are more "natural" MPE where a sophisticate, even under IC, will never be more prone than a naif to develop an addiction. We will address this issue in the following sections.

Concerning part 2, notice that the counterintuitive result vanishes: a sophisticate will never be more prone than a naif to develop an addiction. However, the ORE is left uncharacterized for $k \in [k^{OR}, k^{HR})$. This characterization is a very complicated task: as O&R point out, for this case "sophisticates' behavior can be quite complicated...In fact, because of these complications sophisticates need not follow a stationary strategy or a cutoff strategy". However, we will provide conditions under which a cutoff strategy is a MPE. We also construct non-cutoff equilibria that seem very natural (even under IC).

Since IC plays such an important role in the O&R results, we close this section by proving part 1 of Proposition 9 which in fact is an immediate consequence of the following Lemma whose proof is provided in the appendix.

Lemma 10 *IC implies* $\forall k, U^{H}(k) \geq U^{(0,1,1,\ldots,1)}(k)$ *for* H *and* $(0,1,1,...,1)$ *paths of arbitrary length* T *.*

To see that Lemma 10 implies $\forall t, k, \alpha^{s}(t,k) = 1$, consider a finite horizon T. In period T a sophisticated will hit independently of his addiction level because the instantaneous utility from hitting is always bigger than the one from refraining and there are obviously no future costs of hitting. Self- $T-1$, knowing that self-T will hit no matter what he does, only has to choose between paths $(1,1)$ and $(0,1)$. But Lemma 10 says that $(1,1)$ is preferred for any prevailing addiction level at period $T-1$ and therefore self- $T-1$ will hit independently of his addiction level. Proceeding by backward induction we obtain that a sophisticate will hit in every period. Since this holds for an arbitrarily path length T, in the limit we obtain that a sophisticate will always hit; i.e. ORE generates path H.

Since the ORE solution proves to be unsatisfactory, we turn now to study other sort of MPE. Because TCs and naifs follow cutoff actions, we begin by studying MPE in cutoff strategies for sophisticates.

III. MPE in Cutoff Strategies (CE)

In this section we will characterize CE, i.e. MPE where all selves play the same cutoff strategy

$$\overline{\alpha}(k) = \begin{cases} 0 & \text{if} \quad k < \overline{k} \\ 1 & \text{if} \quad k \geq \overline{k} \end{cases}$$

Lemma 11 *for* $\overline{k} = 0$, $\overline{\alpha}(k)$ *is a CE if and only if IC holds.*

Proof. Suppose $\overline{\alpha}(k)$ is a CE and consider an unaddicted self deviating to some strategy prescribing $\alpha(0) = 0$. The path generated by deviating is $(0,1,1,...)$ while the path generated by sticking is H. For the deviation to be non-profitable we need $U^{H}(0) \geq U^{011\cdots}(0)$, i.e. IC must hold. Now suppose IC holds. Then $U^{H}(0) \geq U^{011\cdots}(0)$ which implies $\forall k, U^{H}(k) \geq U^{011\cdots}(k)$ and therefore no deviation from $\overline{\alpha}(k)$ is profitable.

Lemma 12 *if $k^{HR} \leq k^{OR}$, $\overline{\alpha}(k)$ with $\overline{k} = k^{HR}$ is a CE and it dominates any other equilibrium.*

Proof. First note that $\forall k < k^{HR}$ the DBP is R whereas $\forall k \geq k^{HR}$ the DBP is H. Therefore, by sticking to strategy $\overline{\alpha}(k)$ each self follows his DBP which proves that it is a dominating equilibrium.

Remark 13 *Since in an intrapersonal game the players are different incarnations of the same individual, we believe that equilibrium selection should be resolved, whenever possible, by a Pareto criterion. If there is not a Pareto dominant equilibrium, at least it should be obvious that a dominated equilibrium should not be played. In the case $k^{HR} \leq k^{OR}$, $\overline{\alpha}(k)$ with $\overline{k} = k^{HR}$ pareto-dominates any other equilibrium so it would be quite unnatural to propose any other solution to this game. However, when IC holds, the O&R solution yields the hitting path : They argue that this is sustainable if each self has the pessimistic beliefs that his future selves will hit no matter what his current action is, thus choosing to hit since the path H yields a higher utility than $(0,1,1,...)$. But why should every self have those pessimistic beliefs when they can coordinate on a dominating equilibrium? We believe this is a major drawback of the ORE.*

In a sense, Lemma 12 says that whenever $k^{HR} \leq k^{OR}$ there are no self-control problems since for any addiction level the DBP for a particular self will be followed by his futures selves. As a consequence, awareness of time-inconsistency is immaterial: the paths followed by a naif and a sophisticate are the same since the solution to (5) at any period and for any addiction level is still optimal at further periods.

We will now characterize CE when $k^{HR} > k^{OR}$ (notice that this implies $k^{HR} > 0$). In what follows we will assume that every self is playing the same cutoff strategy $\overline{\alpha}$. We will denote by $V(\overline{\alpha}(k))$ the utility obtained by a self with addiction level k when sticking to $\overline{\alpha}$, whereas $V(\alpha(k))$ will denote his utility when deviating to a particular strategy α while all other selves stick to $\overline{\alpha}$. Throughout we will assume $\overline{k} > 0$ since the case $\overline{k} = 0$ has already been covered in Lemma 11.

Claim 14 *If $\overline{\alpha}$ is a CE then $\overline{k} = k^{HR}$.*

Proof. Suppose $\overline{k} > k^{HR}$ and consider an addiction level $k \in \left(k^{HR}, k^{OR}\right)$. Any strategy α prescribing $\alpha(k) = 1$ is a profitable deviation since $V(\overline{\alpha}(k)) = U^R(k) \leq U^H(k) = V(\alpha(k))$. Now suppose $\overline{k} < k^{HR}$ and consider an addiction level k such that $\gamma k < \overline{k} < k < k^{HR}$ (such a k exists since $\overline{k} > 0$). Any strategy α prescribing $\alpha(k) = 0$ is a profitable deviation since $V(\overline{\alpha}(k)) = U^H(k) \leq U^R(k) = V(\alpha(k))$.

Lemma 15 *If $k^{HR} > k^{OR}$, strategy $\overline{\alpha}(k)$ with $\overline{k} = k^{HR}$ is a CE if and only if $\gamma k^{OR} + 1 \geq k^{HR}$.*

Proof. Take any $k \geq k^{HR}$ and consider deviating to a strategy α prescribing $\alpha(k) = 0$. The path generated by this deviation is either R or $(0, 1, 1, ...)$. If R is generated, the deviation is non profitable since $V(\alpha(k)) = U^R(k) \leq U^H(k) = V(\overline{\alpha}(k))$. Now consider $(0, 1, 1, ...)$ being generated. In the appendix we prove that $U^{(0,1,1,...)}(k) \leq U^H(k)$ for any $k \geq k^{OR}$. Since this is the case we have $V(\alpha(k)) = U^{(0,1,1,...)}(k)$ $\leq U^H(k) = V(\overline{\alpha}(k))$and therefore the deviation is non-profitable. For any $k < k^{OR}$ there is no profitable deviation since sticking to strategy $\overline{\alpha}$ generates the DBP. So consider any $k \in \left[k^{OR}, k^{HR}\right)$. If $\gamma k^{OR} + 1 \geq k^{HR}$, a deviation to any strategy α prescribing $\alpha(k) = 1$ generates path H thus being non profitable since $V(\alpha(k)) = U^H(k) \leq U^R(k) = V(\overline{\alpha}(k))$. This proves $\gamma k^{OR} + 1 \geq k^{HR}$ implies $\overline{\alpha}$ is a CE. Now suppose $\gamma k^{OR} + 1 < k^{HR}$. Take any $k \in \left(k^{OR}, k^{HR}\right)$ such that $\gamma k + 1 < k^{HR}$ and consider a deviation to a strategy α prescribing $\alpha(k) = 1$. The deviation is profitable since it generates path O and $V(\alpha(k)) = U^O(k) > U^R(k) = V(\overline{\alpha}(k))$.

Remark 16 *Suppose IC does not hold and $\gamma k^{OR} + 1 < k^{HR}$. Under these circumstances O&R claim that for $k \in \left[k^{OR}, k^{HR}\right)$ "sophisticates need not follow a cutoff strategy" . Here we go further since Lemma 11 and Lemma 15 imply that under these circumstances sophisticates* ***cannot*** *follow a cutoff strategy because there is no CE. Also notice that if IC holds and $\gamma k^{OR} + 1 \geq k^{HR} > k^{OR}$ there are exactly two CE : the one with $\overline{k} = 0$ and the one with $\overline{k} = k^{HR}$. The latter clearly dominates the former so one would expect the different selves to coordinate in the second equilibrium while O&R take as solution to the game the first one.*

We summarize our results concerning CE in the following Proposition.

Proposition 17 *Characterization of CE and comparison with ORE.*

1. If IC holds then
   (a) If $\gamma k^{OR} + 1 \geq k^{HR}$ then $\overline{\alpha}(k)$ with $\overline{k} = k^{HR}$ and ORE are the only CE. $\overline{\alpha}(k)$dominates ORE.
   (b) If $\gamma k^{OR} + 1 < k^{HR}$ then ORE is the unique CE.
2. If IC does not hold then
   (a) If $\gamma k^{OR} + 1 \geq k^{HR}$ then $\overline{\alpha}(k)$ with $\overline{k} = k^{HR}$ is the unique CE. $\overline{\alpha}(k)$ and ORE coincide.
   (b) If $\gamma k^{OR} + 1 < k^{HR}$ then there exists no CE.

If we are to restrict our attention to CE, the case $\gamma k^{OR} + 1 < k^{HR}$ is a problematic one. If IC holds then we have the ORE solution which proved to be counterintuitive since sophistication could exacerbate over-consumption. If IC doesn't hold then we may have non-existence of CE. This leads us to study non-cutoff equilibria which is done in the following section.

IV. Non-cutoff Equilibria: some examples

In what follows, we still restrict ourselves to MPE where the strategies involved are time-independent. We also focus on the case $\gamma k^{OR} + 1 < k^{HR}$ since the opposite has already been covered.

A. IC holds: "I won't hit because if I do it I will do it forever"

Here we go one step further in resolving IC since we provide a MPE that dominates ORE whenever IC holds.

Lemma 18 *strategy* $\widehat{\alpha}(k) = \begin{cases} 0 & \textit{if} \quad k = 0 \\ 1 & \textit{if} \quad k > 0 \end{cases}$ *is a MPE.*

Proof. For $k = 0$, sticking to $\widehat{\alpha}$ generates path R while deviating generates path H. Since $0 \leq k^{OR} < k^{RH}$ the deviation is non profitable. For any $k > 0$ a deviation is neither profitable since it generates path $(0, 1, 1, ...)$ which is clearly dominated by H (Lemma 10).

Remark 19 *$\widehat{\alpha}$ dominates the ORE though it does it strictly only for $k = 0$: when sticking to $\widehat{\alpha}$ an unaddicted sophisticate will never develop an addiction, moreover any self is strictly better-off by sticking to $\widehat{\alpha}$ since it generates his DBP. However, $\widehat{\alpha}$ has the shortcoming of being non robust or fragile, since any small deviation by any self develops the addiction. Nevertheless, we claim that this equilibrium deserves attention since it captures strategic decisions often observed in the realm of harmful addictive drugs: Decisions of the sort "I won't do it because if I do it I will do it forever" (think of drugs such as heroin).*

B. "Take a walk on the wild side"

We already know that a CE fails to exist if IC is not satisfied and is equal to the ORE otherwise (thus being dominated by naive behavior).
Let $\overline{k} = \frac{k^{HR-1}}{\gamma}$, so that hitting with any $k \geq \overline{k}$ drives the addiction level above k^{HR}. Consider each self following strategy

$$\alpha^{wws}(k) = \begin{cases} 0 & \text{if} \quad k < k^{OR} \\ 1 & \text{if} \quad k^{OR} \leq k < \overline{k} \\ 0 & \text{if} \quad \overline{k} \leq k < k^{HR} \\ 1 & \text{if} \quad k \geq k^{HR} \end{cases}$$

which tries to capture the following idea illustrated in Figure 2:

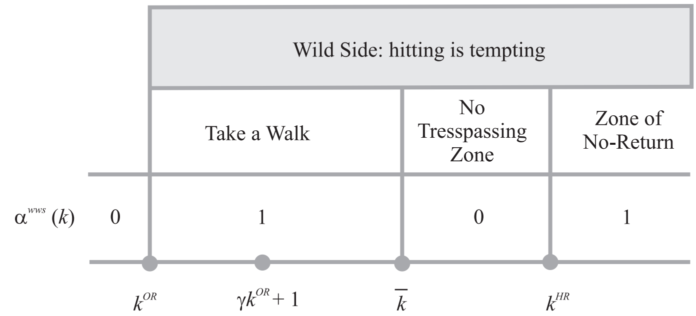


Figure 2: The “Take a Walk on the Wild Side” strategy.

There is a *wild side* ($k \geq k^{OR}$) in which hitting is an effective temptation (for $k < k^{OR}$ hitting is not effectively tempting because always refrain is the DBP) and therefore the individual would like to hit (take a walk). In this *wild side* there is a *zone of no-return* ($k \geq k^{HR}$) since once an individual falls in it he becomes irremediably addicted (he hits forever after). Now notice that hitting leads to the zone of no-return if and only if $k \geq \overline{k}$, therefore $\overline{k}$ marks the point where a *no-trespassing zone* ($\overline{k} < k < k^{HR}$) begins. With strategy α^{wws} the individual hits whenever on the wild side and outside the no-trespassing zone, i.e. he takes a walk but knows when to stop.

Unfortunately, α^{wws} does not always constitute an equilibrium. A counterexample is given in the appendix (Example A, Section VIII) . However, we will provide some conditions under which α^{wws} happens to be a MPE.

Lemma 20 *if* $\gamma k^{HR} < k^{OR} < \overline{k} < \gamma\left(\gamma k^{OR} + 1\right) + 1$ *then* α^{wws} *is a MPE. (This Lemma is illustrated in Figure 3. In the appendix we provide examples satisfying the condition stated with the IC holding, Example B, Section VIII, and not holding, Example C, Section VIII)*

Proof. For any addiction level k such that $k < k^{OR}$ or $k \geq k^{HR}$, α^{wws} generates the respective DBP so there is no profitable deviation. For any k such that $k^{OR} \leq k < k^{HR}$, by sticking to α^{wws} the DBP is generated (and thus there is no profitable deviation). To see this first notice that the DBP is hitting once. Following α^{wws} the individual hits and then drives the addiction level above $\overline{k}$ but below k^{HR}, therefore his immediate future self will refrain and, since by doing so he drives the addiction level below k^{OR}, all other future selves will refrain as well. For any k such that $\overline{k} \leq k < k^{HR}$, α^{wws} generates path R, while any deviation generates path H. Since R is preferred to H we conclude that there is no profitable deviation.

We tackle now the case $k^{HR} \leq k < k^{HO}$. First notice that necessarily $k^{HO} = \frac{k^{tc}-1}{\gamma}$. Let $k_0 = k^{HO}$, define $k_1 = \frac{k_0-1}{\gamma}$ and suppose the current self has addiction level $k \in [k_1, k_0) \cap \left[k^{HR}, k_0\right)$. Clearly, by sticking to α^{wws} hitting with addiction level k generates path H . A deviation, i.e. refraining with addiction level k, drives next-period's addiction level below $\gamma k_0 = k^{tc} - 1 < k^{tc}$ so from the current self's perspective the best possible behavior path following restraint is R (because he would like to behave like a TC from next period on and a TC would like to refrain always for addiction levels below k^{tc}). That is, the best possible behavior path that could be generated by a deviation is R. Since for addiction level k, H is preferred to R we conclude that there is no profitable deviation. But proceeding by induction, the same logic applies for any $k \in [k_{i+1}, k_i) \cap \left[k^{HR}, k_0\right)$ where $k_{i+1} = \frac{k_i-1}{\gamma}$ $(i = 0, 1, ...)$ so we finally conclude that there is no profitable deviation for any k such that $k^{HR} \leq k \leq k^{HO}$.

	R > O > H	O > R > H		O > H > R	H > O > R
$\alpha^{wws}(k)$	0	1	0	1	1
Stick	000...	100...	000...	111...	111...
	DBP	DBP		BPP	DBP
Deviate			111...		
	NPD	NPD	NPD	NPD	NPD

γk^{HR} k^{OR} $\overline{k}$ k^{HR} k^{HO}

In the first row we rank the paths R,H and O. In the second row weput the actions prescribed by the strategy. The third row establishesthe path generated by sticking to the strategy and the fourth establisheswhether this path is the desired behavior path (DBP), the best possiblepath (BPP) or none. The fifth row establishes the path generated bydeviating from the strategy proposed in the second row. Finally, thesixth row establishes whether there is no profitable deviation (NPD).

Figure 3: If $\gamma k^{HR} < k^{OR} < \overline{k} < \gamma\left(\gamma k^{OR} + 1\right) + 1$ then α^{wws} is a MPE.

Remark 21 *If IC does not hold (Example C, Section VIII) we know that there is no cutoff equilibrium,* α^{wws} *proves existence of a non-cutoff MPE. If IC holds (Example B, Section VIII), we know that the unique cutoff equilibrium is hitting always, then* α^{wws} *constitutes an equilibrium that clearly dominates it, moreover,* α^{wws} *also dominates the fragile equilibrium* $\widehat{\alpha}$ *given in Lemma 18.*

V. Developing an Addiction: $k^{HR} > k^{OR} = 0$

O&R state (page 4) "While Becker and Murphy (1988) argue it can be optimal for a person to *maintain* a severely harmful addiction, their steady-state model provides no formal analysis of why the person would choose to *develop* this harmful addiction in the first place." If we are to study why a person could choose to develop an addiction the pertinent starting addiction level must be $k = 0$; i.e. we must focus on the behavior of an unaddicted person. Suppose $k^{OR} > 0$. If $k^{HR} \geq k^{OR}$ we are in a situation as the one depicted in Figure 4.

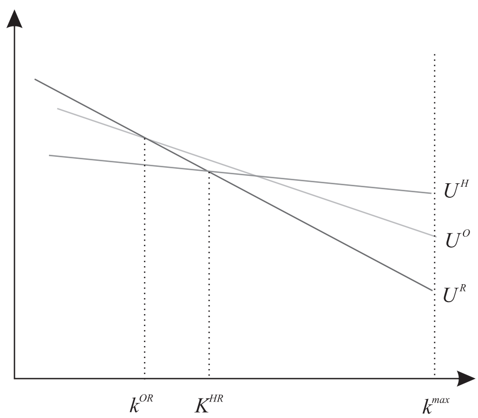


Figure 4: $k^{HR} \geq k^{OR}$

In this case the DBP for an unaddicted person is clearly R, and therefore the addiction will never be developed since the addictive product is not "tempting": no incarnation wants to consume it. If $k^{HR} < k^{OR}$

we should distinguish two cases; $k^{HR} > 0$ and $k^{HR} = 0$; which are illustrated in Figure 5.

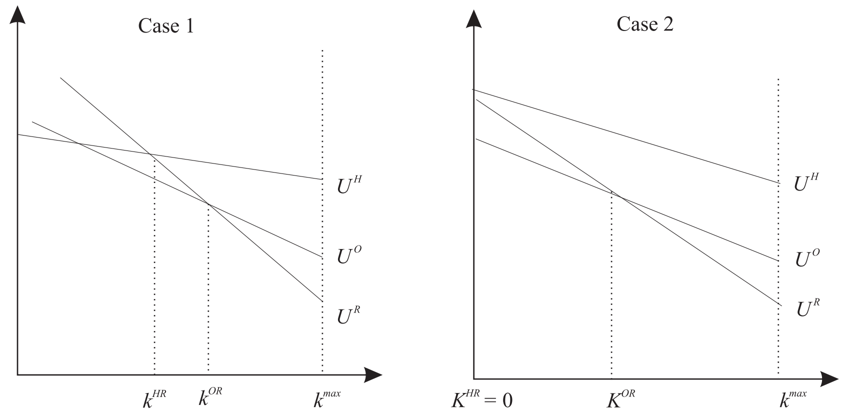


Figure 5: When $k^{HR} < k^{OR}$ we distinguish two cases; $k^{HR} > 0$ and $k^{HR} = 0$

In Case 1 we also obtain the non-tempting condition that ensures that the addiction will never be developed. In Case 2, the DBP for an unaddicted person is H and the addiction is developed (every self will decide to hit). However, in this case there are no self-control problems: each self is following his DBP which amounts to saying that the person is a "happy addict" in the sense that each incarnation behaves precisely as the previous selves desired; each self wants to consume and wants his future selves to consume as well. Therefore, the interesting case (the one presenting self-control problems) for studying the development of an addiction must involve $k^{HR} > k^{OR} = 0$ as depicted in Figure 6. (the case $k^{HR} = k^{OR} = 0$ is similar to Case 2).

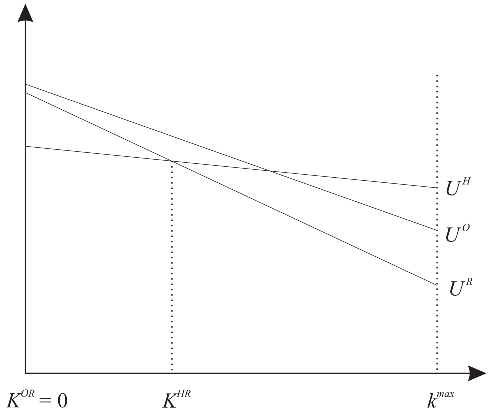


Figure 6: $k^{HR} > k^{OR} = 0$

In this case the unaddicted person would like to hit just once (his DBP is O). If the person is a Naif, he will clearly develop the addiction. We turn now to study a sophisticate's behavior.

First notice that IC holds which implies that in the ORE a sophisticate will also develop an addiction. Are there other equilibria in which a sophisticate does not develop an addiction? The answer is yes since the strategy "I won't hit because if I do it I will do it forever" provided in the previous section is clearly a MPE that induces the refraining path and therefore dominates the ORE. The following Lemma shows an equilibrium that dominates the ORE and generates the hitting once path.

Lemma 22 *Suppose $k^{HR} \geq 1$ and consider strategy*

$$\alpha(k) = \begin{cases} 1 & \textit{if} \quad k = 0 \\ 0 & \textit{if} \quad k = \gamma^i \textit{ for some } i \in \{0, 1, 2, ...\} \\ 1 & \textit{if} \quad \textit{otherwise} \end{cases}$$

This strategy is a MPE that generates path O.

Proof. When $k = 0$, sticking to $\alpha(k)$ generates path O which is the DBP, therefore there is no profitable deviation. For any $k = \gamma^i$ with $i \in \{0, 1, 2, ...\}$, sticking to $\alpha(k)$ generates path R. Now fix i and notice that $\gamma(\gamma^i) + 1 > \gamma^j$ for any $j \in \{0, 1, 2, ...\}$, therefore deviating from $\alpha(k)$ generates path H which is non-profitable. For any other k sticking to $\alpha(k)$ generates path H (because for any $j \in \{0, 1, 2, ...\}$, $\gamma k + 1 > \gamma^j$) while deviating generates path $(0, 1, 1, ...)$ (because if $k \neq \gamma^i$ for all $i \in \{0, 1, 2, ...\}$, then $\gamma k \neq \gamma^j$ for all $j \in \{0, 1, 2, ...\}$) which is non-profitable since IC holds.

This equilibrium dominates the ORE, but compared to the strategy "I won't hit because if I do it I will do it forever" the initial self (unaddicted person) is strictly better-off while any future self is strictly worse-off.

VI. Discussion

The O&R set-up seems appropriate for modeling addiction since it incorporates the two basic features of an addictive substance (habit-formation and negative internalities) and it allows for self-control problems which have been largely documented in the psychological literature. This is an improvement with respect to the Becker-Murphy model of rational addiction in which self-control problems were inexistent. However, their particular equilibrium selection (ORE) for the intrapersonal game induced by sophisticated behavior has the shortcoming of producing a counterintuitive result: awareness of self-control problems may exacerbate over-consumption.

We have shown that this paradox vanishes when considering other sort of equilibria that dominate the ORE and that seem more natural since they capture behaviors often observed in the realm of addiction. Since in an intrapersonal game the players are incarnations of the same individual, coordination on a dominated equilibrium is hard to sustain[5]. This favors our equilibria over the ORE and therefore we readily obtain that naifs are more prone to become addicted than sophisticates. The

[5] Carrillo and Mariotti (2000) obtain a similar conclusion. In their model the consumer is uncertain about the degree of addictiveness of the product but may acquire free information which eventually reveals the true degree. In the finite-horizon version each self decides to acquire the information, therefore, when considering the equilibrium of the infinite horizon case that is the limit of the finite case equilibrium, each self decides to get fully informed. But there are other dominating equilibria (refered to as strategic ignorance equilibria) where the selves decide not to get fully informed.

only cases where the ORE is a dominating equilibrium (and therefore it is the appropriate solution concept to be called upon) is when the desired behavior path is to consume always or when IC does not hold and $\gamma k^{OR} + 1 \geq k^{HR}$.

Another advantage of the O&R set up over the Becker-Murphy model is that it permits to explain why an unaddicted person could decide to consume and develop an addiction. We have seen that for this to be possible consumption should be tempting (in the sense that the desired behavior path cannot be refraining) in which case a naïf will always become addicted. Regarding sophisticate behavior, we proved that when the DBP is either hit always or hit once then the inevitability condition must hold and therefore the ORE implies that a sophisticate will also become addicted. This makes sense only when the DBP is hit always since in this case there are no self-control problems and thus naïf and sophisticate behavior coincide. But when the DBP is hitting once, we provide equilibria where a sophisticate will not become addicted which copes with the general view that addiction is the outcome of naïve behavior.

Naiveness and sophistication are extreme degrees of awareness and one would expect that real-world behaviors lie somewhere in between. We want to conclude by suggesting a way to model partial awareness. Very little has been done in this direction: O'Donoghue and Rabin (2001a) formulate an approach to partial naivete in which a partially naïf agent is simply a sophisticate who overestimates his present-biased parameter β. O'Donoghue and Rabin (2001b) propose an approach to boundedly rational incomplete awareness in which agents "don't do all the rounds of backwards-induction. In other words, instead of starting the backwards-induction logic in the last period, they might start the process, say, three periods hence.". We believe that the first approach is somehow ad-hoc while the second is not applicable to the infinite horizon case since it relies on the backwards-induction logic. We suggest a very natural approach: people are initially naïf and as time elapses they become aware of their self-control problems (i.e. they become sophisticates). This approach is also suggested by Elster (1999): "reversal experiences can give rise to learning. Once the person observes himself reversing his decisions time and again, he will come to know that this is just the way he behaves under these circumstances. In the language of O'Donoghue and Rabin, he is no longer naive, but sophisticated." Obviously there would be persons that become aware more quickly than others; to be more precise, we could define an agent as a t-naif when it takes him t periods to become aware of his time inconsistency. With this formulation, naifs and sophisticates are ∞-naifs and 0-naifs respectively. This would allow to observe behaviors which imply hitting for a finite number of times: an example is provided in the appendix.

VII. Appendix - Proofs

A. Proposition 5

Proof.

1. $U^A(k)$ is decreasing. Let $k_t(A,k)$ be the addiction level prevailing at time t conditional on following path A with starting addiction level k; i.e. $k_1(A,k)=k$; $k_t(A,k)=\gamma^{t-1}k+\sum_{i=1}^{t-1}\gamma^{t-i-1}a_i$ for $t=2,3,...$ Then

$$U^A(k)=\begin{bmatrix} a_1(x+f(k))+ \\ (1-a_1)g(k) \end{bmatrix}+\beta\sum_{t=2}^{\infty}\delta^{t-1}\begin{bmatrix} a_t(x+f(k_t(A,k)))+ \\ (1-a_t)g(k_t(A,k)) \end{bmatrix}$$

and therefore

$$\frac{\partial U^A(k)}{\partial k}=\begin{bmatrix} a_1 f'(k)+ \\ (1-a_1)g'(k) \end{bmatrix}+\beta\sum_{t=2}^{\infty}\delta^{t-1}\begin{bmatrix} a_t\gamma^{t-1}f'(k_t(A,k))+ \\ (1-a_t)\gamma^{t-1}g'(k_t(A,k)) \end{bmatrix}$$

which is equal to

$$\begin{bmatrix} a_1 f'(k)+ \\ (1-a_1)g'(k) \end{bmatrix}+\beta\delta\sum_{t=1}^{\infty}\delta^{t-1}\gamma^t\begin{bmatrix} a_{t+1}f'\left(\gamma^t k+\sum_{i=1}^{t}\gamma^{t-i}a_i\right)+ \\ (1-a_{t+1})g'\left(\gamma^t k+\sum_{i=1}^{t}\gamma^{t-i}a_i\right) \end{bmatrix} \tag{1}$$

and because of negative internalities (Assumption 1: $f', g' < 0$) we readily obtain $\frac{\partial U^A(k)}{\partial k}\leq 0$.

2. $\forall k$, $\frac{\partial U^H(k)}{\partial k} \geq \frac{\partial U^A(k)}{\partial k} \geq \frac{\partial U^R(k)}{\partial k}$. Since $U^H(k) = x\left[\frac{1-\delta+\beta\delta}{1-\delta}\right] + f(k) + \beta\delta\sum_{t=1}^{\infty}\delta^{t-1}f\left(\gamma^t k + \sum_{i=0}^{t-1}\gamma^i\right)$we have

$$\frac{\partial U^H(k)}{\partial k} = f'(k) + \beta\delta\sum_{t=1}^{\infty}\delta^{t-1}\gamma^t f'\left(\gamma^t k + \sum_{i=0}^{t-1}\gamma^i\right) \qquad (2)$$

from (1) and (2) we can express $\frac{\partial U^H(k)}{\partial k} - \frac{\partial U^A(k)}{\partial k}$ as the sum of the following terms

$$(1-a_1)\left(f'(k) - g'(k)\right) \qquad (3)$$

$$\beta\delta\sum_{t=1}^{\infty}\delta^{t-1}\gamma^t a_{t+1}\left[\begin{array}{c} f'\left(\gamma^t k + \sum_{i=0}^{t-1}\gamma^i\right) - \\ f'\left(\gamma^t k + \sum_{i=1}^{t}\gamma^{t-i}a_i\right)\end{array}\right] \qquad (4)$$

$$\beta\delta\sum_{t=1}^{\infty}\delta^{t-1}\gamma^t(1-a_{t+1})\left[\begin{array}{c} f'\left(\gamma^t k + \sum_{i=0}^{t-1}\gamma^i\right) - \\ g'\left(\gamma^t k + \sum_{i=1}^{t}\gamma^{t-i}a_i\right)\end{array}\right] \qquad (5)$$

(3) is positive because of the habit-forming feature (Assumption 2: $f' - g' > 0$); (4) is positive because $\gamma^t k + \sum_{i=0}^{t-1}\gamma^i > \gamma^t k + \sum_{i=1}^{t}\gamma^{t-i}a_i$ and $f'' \geq 0$ (Assumption 3) imply

$$f'\left(\gamma^t k + \sum_{i=0}^{t-1}\gamma^i\right) - f'\left(\gamma^t k + \sum_{i=1}^{t}\gamma^{t-i}a_i\right) \geq 0 \qquad (6)$$

(5) is positive because (6) and $f' - g' > 0$ imply

$$\begin{aligned} 0 \ < \ & f'\left(\gamma^t k + \sum_{i=1}^{t}\gamma^{t-i}a_i\right) - g'\left(\gamma^t k + \sum_{i=1}^{t}\gamma^{t-i}a_i\right) \leq \\ & f'\left(\gamma^t k + \sum_{i=0}^{t-1}\gamma^i\right) - g'\left(\gamma^t k + \sum_{i=1}^{t}\gamma^{t-i}a_i\right)\end{aligned}$$

therefore we have

$$\forall k, \frac{\partial U^H(k)}{\partial k} - \frac{\partial U^A(k)}{\partial k} \geq 0 \qquad (7)$$

Since $U^R(k) = g(k) + \beta\delta\sum_{t=1}^{\infty}\delta^{t-1}g(\gamma^t k)$ we have

$$\frac{\partial U^R(k)}{\partial k} = g'(k) + \beta\delta\sum_{t=1}^{\infty}\delta^{t-1}\gamma^t g'\left(\gamma^t k\right) \qquad (8)$$

from (1) and (8) we can express $\frac{\partial U^A(k)}{\partial k} - \frac{\partial U^R(k)}{\partial k}$ as the sum of the following terms

$$a_1 \left(f^{'}(k) - g^{'}(k) \right) \tag{9}$$

$$\beta\delta \sum_{t=1}^{\infty} \delta^{t-1} \gamma^t (1 - a_{t+1}) \left[\begin{array}{c} g^{'}\left(\gamma^t k + \sum_{i=1}^t \gamma^{t-i} a_i\right) - \\ g^{'}\left(\gamma^t k\right) \end{array} \right] \tag{10}$$

$$\beta\delta \sum_{t=1}^{\infty} \delta^{t-1} \gamma^t a_{t+1} \left[\begin{array}{c} f^{'}\left(\gamma^t k + \sum_{i=1}^t \gamma^{t-i} a_i\right) \\ -g^{'}\left(\gamma^t k\right) \end{array} \right] \tag{11}$$

(9) is positive by the habit-forming feature; (10) is positive because $\gamma^t k + \sum_{i=1}^t \gamma^{t-i} a_i \geq \gamma^t k$ and $g^{''} \geq 0$ (Assumption 3) imply

$$g^{'}\left(\gamma^t k + \sum_{i=1}^{t} \gamma^{t-i} a_i\right) \geq g^{'}\left(\gamma^t k\right) \tag{12}$$

and (11) is positive because (12) and $f' - g' > 0$ imply

$$0 \leq g^{'}\left(\gamma^t k + \sum_{i=1}^{t} \gamma^{t-i} a_i\right) - g^{'}\left(\gamma^t k\right) \leq f^{'}\left(\gamma^t k + \sum_{i=1}^{t} \gamma^{t-i} a_i\right) - g^{'}\left(\gamma^t k\right) \tag{13}$$

therefore we have

$$\forall k, \frac{\partial U^A(k)}{\partial k} - \frac{\partial U^R(k)}{\partial k} \geq 0 \tag{14}$$

combining (7) and (14) completes the proof.

B. Claim used in Lemma 15

Claim 23 *$\exists k^{'}$ such that $U^H(k) \geq U^{(0,1,1,\ldots)}(k) \Leftrightarrow k \geq k^{'}$. Moreover, $k^{'} \leq k^{OR}$.*

Proof. If IC holds, we obtain trivially $k^{'} = 0 \leq k^{OR}$. So assume that IC doesn't hold, that is, $U^H(0) < U^{(0,1,1,\ldots)}(0)$. Define $\Delta(k) = U^H(k) - U^{(0,1,1,\ldots)}(k)$. Simple calculations yield

$$\Delta(k) = x + f(k) - g(k) + \beta\delta \sum_{n=1}^{\infty} \delta^{n-1} \left[\begin{array}{c} f\left(\gamma^n k + \sum_{i=0}^{n-1} \gamma^i\right) - \\ f\left(\gamma^n k + \sum_{i=0}^{n-2} \gamma^i\right) \end{array} \right]$$

Now notice that $\Delta(k)$ is continuous and increasing in k (by $f' - g' < 0$ and convexity of f). Since $\Delta(0) < 0$ (because IC doesn't hold) and $\Delta(k^{\max}) \geq 0$ such a k' exists, and we must have $\Delta\left(k'\right) = 0$ which can be rewritten as

$$x + f\left(k'\right) - g\left(k'\right) = \beta\delta \sum_{n=1}^{\infty} \delta^{n-1} \left[\begin{array}{c} f\left(\gamma^n k' + \sum_{i=0}^{n-2} \gamma^i\right) - \\ f\left(\gamma^n k' + \sum_{i=0}^{n-1} \gamma^i\right) \end{array} \right] \quad (15)$$

Suppose now $k' > k^{OR}$. By definition of k^{OR} we must have $U^O\left(k'\right) > U^R\left(k'\right)$ which is equivalent to

$$x + f\left(k'\right) - g\left(k'\right) > \beta\delta \sum_{n=1}^{\infty} \delta^{n-1} \left[\begin{array}{c} g\left(\gamma^n k'\right) - \\ g\left(\gamma^n k' + \gamma^{n-1}\right) \end{array} \right] \quad (16)$$

Substracting (15) from (16) we obtain

$$0 > \beta\delta \sum_{n=1}^{\infty} \delta^{n-1} \left[\begin{array}{c} g\left(\gamma^n k'\right) - g\left(\gamma^n k' + \gamma^{n-1}\right) + \\ f\left(\gamma^n k' + \sum_{i=0}^{n-1} \gamma^i\right) - f\left(\gamma^n k' + \sum_{i=0}^{n-2} \gamma^i\right) \end{array} \right] \quad (17)$$

by convexity of g we have

$$g\left(\gamma^n k'\right) - g\left(\gamma^n k' + \gamma^{n-1}\right) \geq g\left(\gamma^n k' + \sum_{i=0}^{n-2} \gamma^i\right) - g\left(\gamma^n k' + \sum_{i=0}^{n-1} \gamma^i\right)$$

which together with (17) yields

$$0 > \beta\delta \sum_{n=1}^{\infty} \delta^{n-1} \left[\begin{array}{c} g\left(\gamma^n k' + \sum_{i=0}^{n-2} \gamma^i\right) - g\left(\gamma^n k' + \sum_{i=0}^{n-1} \gamma^i\right) + \\ f\left(\gamma^n k' + \sum_{i=0}^{n-1} \gamma^i\right) - f\left(\gamma^n k' + \sum_{i=0}^{n-2} \gamma^i\right) \end{array} \right]$$

which in turn can be rewritten as

$$0 > \beta\delta \sum_{n=1}^{\infty} \delta^{n-1} \left[h\left(\gamma^n k' + \sum_{i=0}^{n-1} \gamma^i\right) - h\left(\gamma^n k' + \sum_{i=0}^{n-2} \gamma^i\right) \right] \quad (18)$$

where $h = f - g$. But this is a contradiction because h is increasing and therefore the RHS of (18) is positive. This completes the proof.

VIII. Appendix - Examples

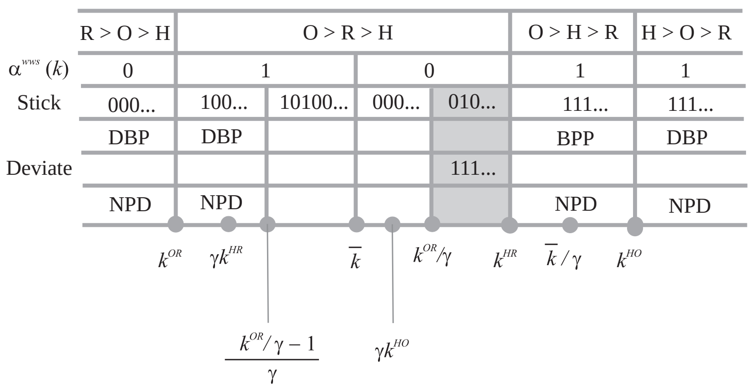
All the numeric examples that follow use the specific instantaneous utility function:

$$\forall t, u\left(k_t, a_t\right) = \begin{cases} x - \rho k_t & \text{if} \quad a_t = 1 \\ -\left(\rho + \sigma\right) k_t & \text{if} \quad a_t = 0 \end{cases}$$

with x, ρ and σ positive, which clearly satisfies the conditions given in Section I.

A. An example where α^{wws} fails to be an equilibrium

When $k^{OR} < \gamma k^{HR} < \overline{k} < \gamma k^{HO} < k^{HR} < \gamma\left(\gamma k^{OR} + 1\right) + 1$, a necessary condition for α^{wws} to be an equilibrium is $U^{(0,1,0,\ldots)}\left(k^{HR}\right) \geq U^{H}\left(k^{HR}\right)$. To see this, consider the following figure:



In the first row we rank the paths R, H and O. In the second row we put the actions prescribed by the strategy,. The third row establishes the path generated by sticking to the strategy and the fourth establishes whether this path is the desired behavior path (DBP), the best possible path (BPP) or none. The fifth row establishes the path generated by deviating from the strategy proposed in the second row. Finally, the sixth row establishes whether there is no profitable deviation (NPD). Consider for example, an addiction level $k \in \left[\frac{k^{OR}}{\gamma}, k^{HR}\right)$. For α^{wws} to be an equilibrium, the path $(0, 1, 0, \ldots)$ must be preferred to H. But this occurs only if $U^{(0,1,0,\ldots)}\left(k^{HR}\right) \geq U^{H}\left(k^{HR}\right)$. Parameter values where this doesn't hold (and therefore α^{wws} fails to be an equilibrium), are

$$\beta = 0.78 \quad \delta = 0.9 \quad \rho = 40.5 \quad \sigma = 20 \quad \gamma = 0.8 \quad x = 102$$

B. An example satisfying $\gamma k^{HR} < k^{OR} < \overline{k} < \gamma\left(\gamma k^{OR} + 1\right) + 1$ and IC

$$\beta = 0.79 \quad \delta = 0.9 \quad \rho = 40 \quad \sigma = 20 \quad \gamma = 0.8 \quad x = 102$$

C. An example satisfying $\gamma k^{HR} < k^{OR} < \overline{k} < \gamma\left(\gamma k^{OR} + 1\right) + 1$ but not IC

$$\beta = 0.9 \quad \delta = 0.9 \quad \rho = 40 \quad \sigma = 20 \quad \gamma = 0.9 \quad x = 101$$

D. $t-$naiveness: an example where the realized behavior path involves hitting a finite number of times

Let $k_0 = k^{OR}$ and define $k_i = \gamma k_{i-1} + 1$ for $i = 1, 2, \ldots$. Let $j = \max\left\{i : k_i \leq k^{HR}\right\}$ so that j is the maximum number of consecutive hits, starting from k^{OR}, that keep the addiction level below k^{HR}. Let $\overline{k}_1 = \frac{k^{HR-1}}{\gamma}$ and define $\overline{k}_i = \frac{\overline{k}_{i-1}-1}{\gamma}$ for $i = 1, 2, \ldots$. Suppose j is odd and $U^{R}\left(\overline{k}_2\right) \geq U^{1100\ldots}\left(\overline{k}_2\right)$, then the following strategy is a MPE:

$$\alpha(k) = \begin{cases} 0 & \text{if } \quad k < k^{OR} \\ 1 & \text{if } \quad k^{OR} \leq k < \overline{k}_j \\ 0 & \text{if } \quad \overline{k}_{2n+1} \leq k < \overline{k}_{2n} \text{ for } 2n+1 < j \\ 1 & \text{if } \quad \overline{k}_{2n} \leq k < \overline{k}_{2n-1} \text{ for } 2n < j \end{cases}$$

Starting from k^{OR}, a sophisticate will hit once, a naif will hit forever and:

A 1-naif will hit once
A 2-naif and a 3-naif will hit 3 times

A 4-naif and a 5-naif will hit 5 times
...
A j-1-naif and a j-naif will hit j times
A n-naif for n bigger than j will hit forever.
An example with $j = 5$ is given by

$$\beta = 0.9 \quad \delta = 0.9 \quad \rho = 43 \quad \sigma = 20 \quad \gamma = 0.9 \quad x = 106$$

References

BECKER, G. S. and MURPHY, K. M. (1988). "A Theory of Rational Addiction", *Journal of Political Economy*, 96:675-700.

BERNHEIM, D. and RANGEL, A. (2002). "Addiction and Cue-conditioned Cognitive Processes. " Mimeo.

CARRILLO, J. and MARIOTTI, T. (2000). "Strategic Ignorance as a Self-Disciplining Device", *Review of Economic Studies*, 67:529-544.

CHALOUPKA, F. J. and WARNER, K. E. (1998). "The economics of smoking", In *Handbook of Health Economics*.

ELSTER, J. (1999). Introduction in Jon Elster, ed., *Addiction: Entries and Exits*. New York: Russell Sage Foundation, ix-xx.

FREDERICK, S., LOEWENSTEIN, G. and O'DONOGHUE, T. (2002). "Time Discounting and Time Preference: A Critical Review," *Journal of Economic Literature*, 40(2):351-401.

GRUBER, J. and KOSZEGI, B. (2001). "Is Addiction 'Rational'? Theory and Evidence," *Quarterly Journal of Economics*, 116:1261-1303.

HEYMAN, G. M. (1996). "Resolving the Contradictions of Addiction", *Behavioral and Brain Sciences*, 19:561-610.

LAIBSON, D. (1994). "Essays in Hyperbolic Discounting", Economics, MIT.

LAIBSON, D. (1997). "Golden Eggs and Hyperbolic Discounting" , *Quarterly Journal of Economics*, 112:443-477.

LAIBSON, D. (2001). "A Cue-Theory of Consumption" , *Quarterly Journal of Economics*, 116(1):81-120.

LOEWENSTEIN, G. and PRELEC, D. (1992). "Anomalies in Intertemporal Choice: Evidence and an Interpretation", *Quarterly Journal of Economics*, 107:573-597.

MESSINIS, G. (1999). "Habit Formation and the Theory of Addiction", *Journal of Economic Surveys*, 13:417-442.

O'DONOGHUE, T. and RABIN, M. (2001a). "Choice and Procrastination" , *Quarterly Journal of Economics*, 116(1):121-160.

O'DONOGHUE, T. and RABIN, M. (2001b). "Self Awareness and Self Control." To appear as a chapter in *Now or Later: Economic and Psychological Perspectives on Intertemporal Choice*, edited by Roy Baumeister, George Loewenstein, and Daniel Read, published by Russell Sage Foundation Press.

O'DONOGHUE, T. and RABIN, M. (2002). "Addiction and Present-Biased Preferences", Mimeo.

PHELPS, E. S. and POLLAK, R. A. (1968). "On Second-best National Saving and Game-equilibrium Growth", *Review of Economic Studies*, 35:185-199.

STROTZ, R. H. (1956). "Myopia and Inconsistency in Dynamic Utility Maximization", *Review of Economic Studies*, 23:165-180.

THALER, R. (1991). "Some Empirical Evidence on Dynamic Inconsistency." in *Quasi Rational Economics*. New York: Russell Sage Foundation, 127-133.